

Activity AO4

1. Solve the equation $2\log_5(2y + 1) - \log_5(2 - y) = 1$

explaining clearly why there is only one real solution

2. Solve the equation

$$\log_4(5x^2 - 11) = \log_2(3x - 5).$$

(x real)

3. In which lines is the mathematical reasoning incorrect?

Solve $\cos 2x + \sin 2x = \frac{1}{\sqrt{2}}$ for values of x in the interval $0 \leq x \leq 2\pi$

(1) $\cos 2x + \sin 2x = \sqrt{2} \cos(2x + \frac{\pi}{4})$

(2) So the equation becomes $\cos(2x + \frac{\pi}{4}) = \frac{1}{2}$

(3) A solution of $\cos \theta = \frac{1}{2}$ is $\theta = \frac{\pi}{3}$

(4) So $2x + \frac{\pi}{4} = \frac{\pi}{3}$ and $x = \frac{\pi}{24}$

(5) The second solution is $2\pi - \frac{\pi}{24} = \frac{23\pi}{24}$

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Answers

1. $\frac{(2y+1)^2}{2-y} = 5$ leading to $4y^2 + 9y - 9 = 0$

This quadratic equation has solutions $y = 3/4, y = -3$

When $y = 3/4$, both $2y + 1$ and $2 - y$ are positive - so a solution to the original equation

When $y = -3$, $2y + 1 = -3$ so the log is not real - so not a solution.

2. Using change of base

$$5x^2 - 11 = (3x - 5)^2 \text{ leading to } 2x^2 - 15x + 18$$

This quadratic equation has solutions $x = 3/2, x = 6$

Checking in the original equation $x = 3/2$ is not a solution as $\log(3x - 5) = \log(-0.5)$ which is not real.

3. Line (1) should be $\cos 2x + \sin 2x = \sqrt{2} \cos(2x - \frac{\pi}{4})$

Line (2) does follow logically from line 1

Line (3) is a correct statement

Line (4) is a correct statement

Line (5) does not follow logically from line(3)and (4)

It should be $2x + \frac{\pi}{4} = \frac{5\pi}{3}$ and $x = \frac{17\pi}{24}$